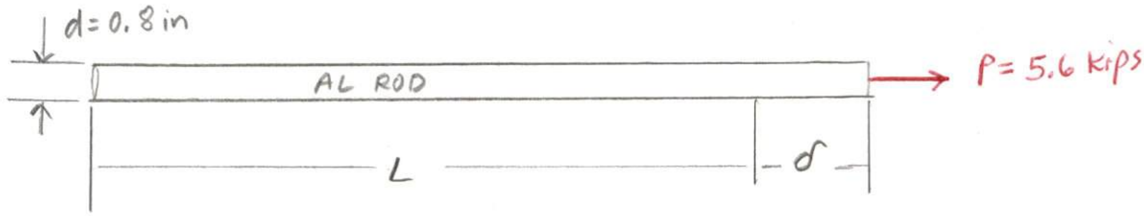
Chapter 10 - Problems

Note: All SI Unit Problems changed to US Customary Units

10-2

An aluminum rod of 0.8 in. (0.20-mm) diameter is elongated 9/64 in. (3.5 mm) along its longitudinal direction by a load of 5.6 kips (25 kN). If the modulus of elasticity of aluminum is E = 10,153 ksi (70 GPa), determine the original length of the bar.



$E = 10{,}153$ ksi (modulus of elasticity of aluminum)

$\delta = 9/64$ in.

Axial Deformation

$$\delta = \frac{PL}{AE}$$

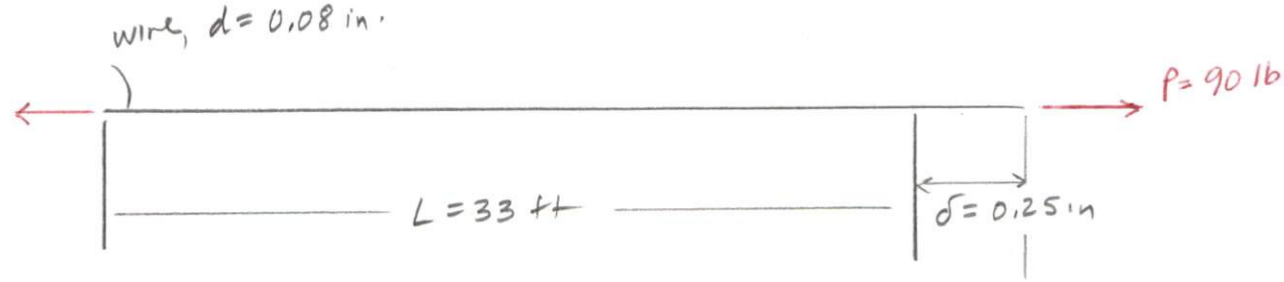
$$L = \frac{AE\delta}{P} = \frac{\frac{\pi (0.8\,in)^2}{4}\left(10{,}153\,\frac{kip}{in^2}\right)\left(9/64\,in.\right)}{5.6\,kip}$$

$$= 128\ in$$

$$= 10.7\ ft$$

10-4

A metal wire is 33 ft (10 m) long and 0.08 in (2 mm) in diameter. It is elongated 0.25 in (6.06 mm) by a tensile force of 90 lb (400 N). Determine the modulus of elasticity of the material and indicate a possible material for the wire.



Axial Deformation

$$\delta = \frac{PL}{AE}$$

$$E = \frac{PL}{\delta A} = \frac{90\ \text{lb}\ (33\ \text{ft})\left(\frac{12\ \text{in}}{\text{ft}}\right)}{0.25\ \text{in}\ \left(\frac{\pi (0.08\ \text{in})^2}{4}\right)}$$

$$= \frac{35640\ \text{lb}\cdot\text{in}}{0.00125 6637\ \text{in.}^3}$$

$$= 28{,}361\ \text{ksi}$$

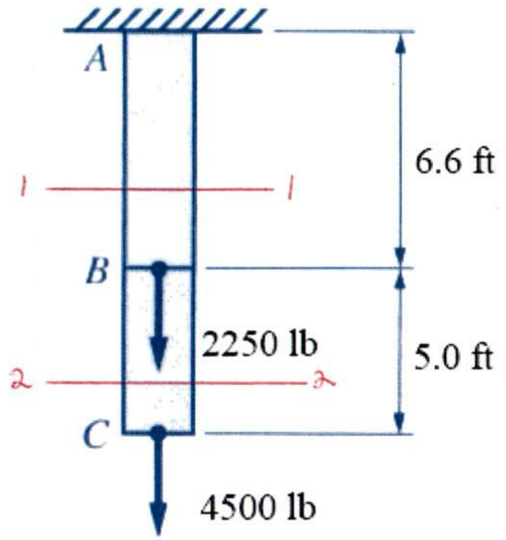
From Table A-7(a)

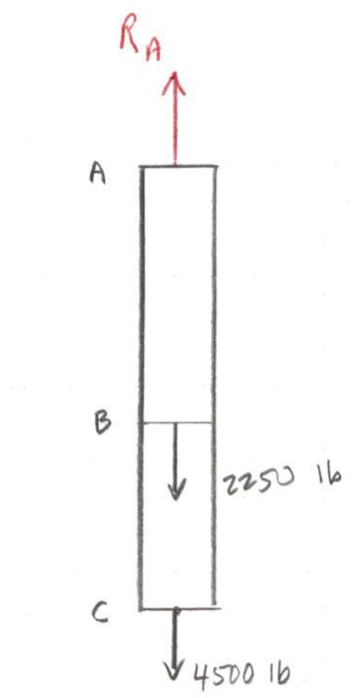
$E = 30{,}000$ ksi Steel

The wire is made of Steel.

10-8

An aluminum bar 1.2 in. (30 mm) in diameter is suspended as shown in Fig. P10-8. Determine the total displacement of the lower end C after the loads are applied. The modulus of elasticity of aluminum is E = 10,153 ksi (70 GPa).



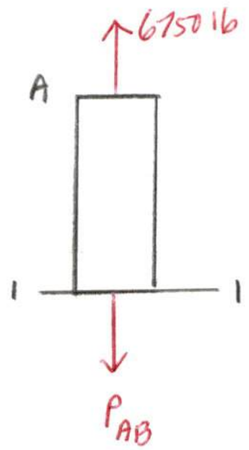


FBD- Entire Bar

Equilibrium Equations

$[\Sigma F_y = 0]$ $\quad R_A - 2250\text{ lb} - 4500\text{ lb} = 0$

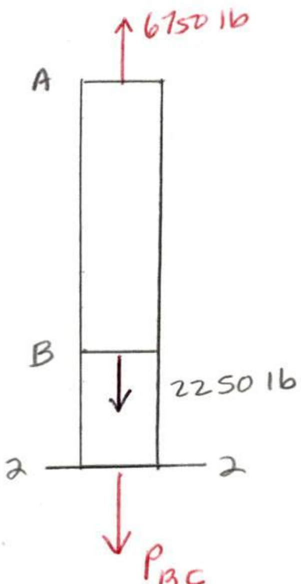
$R_A = 6750\text{ lb} \uparrow$



Equilibrium Equations

$\Sigma F_y = 0$ $\quad P_{AB} = 6750 \text{ lb (T)}$

FBD - Upper Portion of Section 1-1



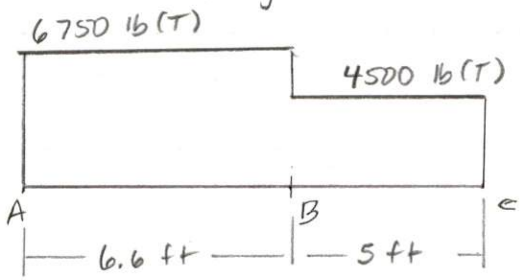
Equilibrium Equations

$[\Sigma F_y = 0] \quad 6750 \text{ lb} - 2250 \text{ lb} - P_{BC} = 0$

$P_{BC} = 4500 \text{ lb (T)}$

FBD - upper Portion of Section 2-2

Axial Force Diagram



The maximum load occurs in Segment AB. The tensile stress in this segment is

$$\sigma = \frac{P}{A} = \frac{6750 \text{ lb}}{\frac{\pi (1.2 \text{ in})^2}{4}} = 5968 \text{ psi} < \sigma_p \text{ aluminum}$$

(Proportional Limit)

$\therefore$ Hooke's Law Applies

$$AE = \pi \frac{(1.2\,in)^2}{4}\left(10{,}153\ ksi\right) = 11{,}483\ kips\left(\frac{1000\ lb}{kip}\right)$$

$$= 11{,}483{,}000\ lb$$

Axial Deformation

$$\delta_{AB} = \frac{P_{AB} L_{AB}}{AE} = \frac{6750\ lb\ (6.6\ ft)}{11{,}483{,}000\ lb} = 0.003876948\ ft$$

$$\delta_{BC} = \frac{P_{BC} L_{BC}}{AE} = \frac{4500\ lb\ (5.0\ ft)}{11{,}483{,}000\ lb} = 0.001959418\ ft$$

Total Deformation

$$\delta_{AC} = \delta_{AB} + \delta_{BC} = 0.003876948\ ft + 0.001959418\ ft$$

$$= 0.005839\ ft \times \left(\frac{12\,in}{ft}\right)$$

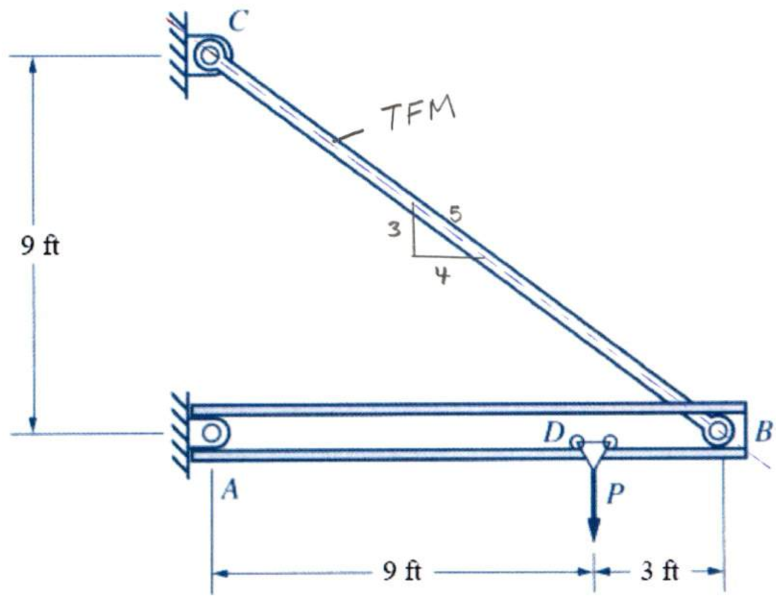
$$= \underline{\underline{0.07}}\ in.\ (elongation)$$

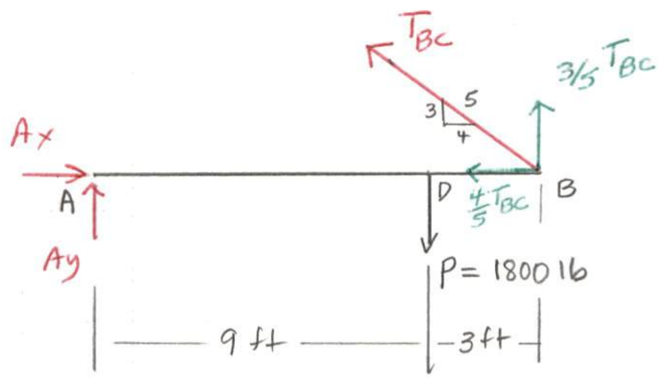
(0.07 in = 1.8 mm)

Check w/text ans ✓

10-10

See Fig. P10-10. Determine the total elongation of the steel eye bar BC of 0.4 in. (10-mm) diameter due to the load P = 1800 lb (8 kN). The modulus of elasticity of steel is E= 30,460 ksi (210 GPa).





ccw +M
cw −M

FBD- Member ADB

Equilibrium Equations

$[\Sigma M_A = 0]$ $\quad - 1800\text{ lb}\,(9\text{ ft}) + 3/5\ T_{BC}\,(12\text{ ft}) = 0$

$$T_{BC} = \frac{\frac{5}{3}(16{,}200\text{ lb}\cdot\text{ft})}{12\text{ ft}} = 2250\text{ lb (T)}$$

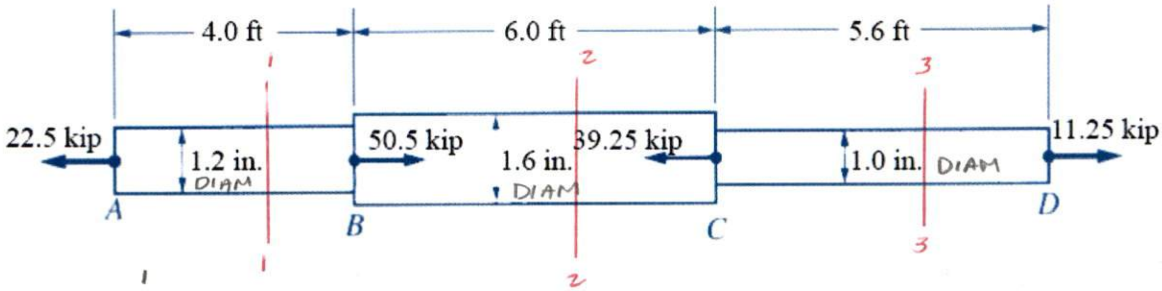
$$A = \frac{\pi d^2}{4} = \frac{\pi(0.4\text{in})^2}{4} = 0.12566\text{ in}^2$$

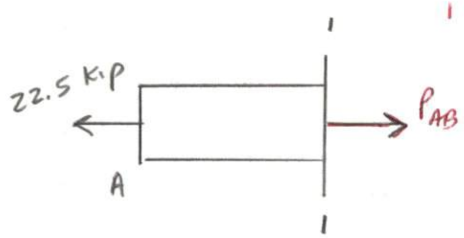
$$\delta = \frac{PL}{AE} = \frac{2250\text{ lb}\,(15\text{ ft})\left(\frac{\text{Kip}}{1000\text{ lb}}\right)}{0.12566\text{ in}^2\,(30{,}460\text{ ksi})} = 0.0088\text{ ft}$$

$$= 0.106\text{ in.}$$

10-11

Determine the total deformation between points A and D of a stepped steel bar subjected to the axial forces shown in Fig. P10-11. The modulus of elasticity of steel is E= 30,460 ksi (210 GPa).



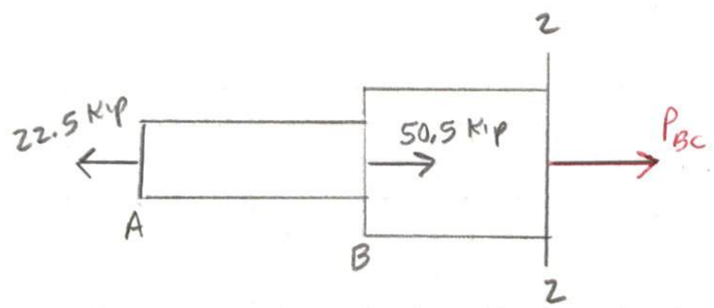


FBD- Left Portion Section 1-1

Equilibrium Equations

$(\Sigma F_x = 0)$ $\quad -22.5 \text{ Kips} + P_{AB} = 0$

$P_{AB} = 22.5 \text{ Kips (T)}$



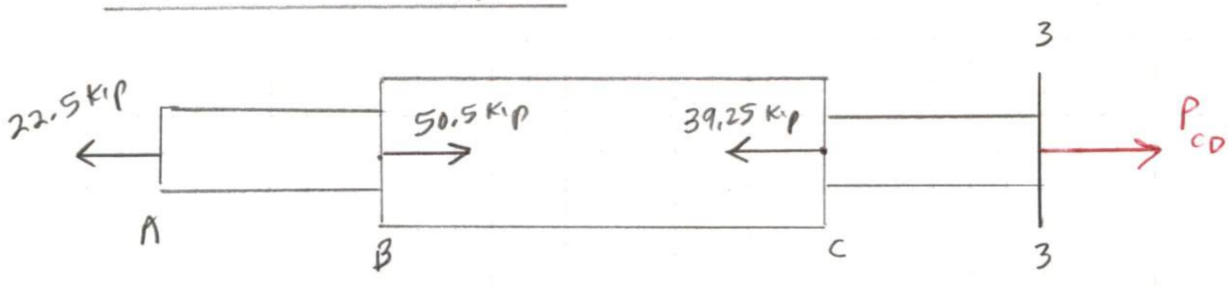
FBD- Left Portion Section 2-2

Equilibrium Equations

$[\Sigma F_x = 0]$ $\quad -22.5 \text{ Kips} + 50.5 \text{ Kips} + P_{BC} = 0$

$P_{BC} = -28 \text{ Kips (T)}$

and $\quad P_{BC} = 28 \text{ Kips (C)}$



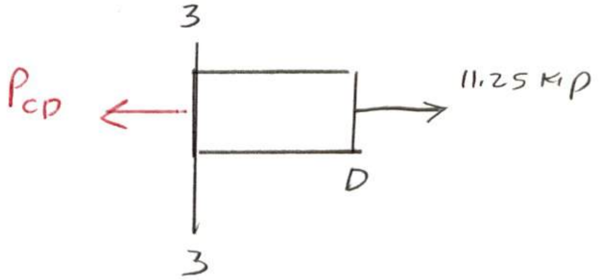
FBD- Left Portion Section 3-3

Equilibrium Equations

$[\Sigma F_x = 0]$ $\quad -22.5 \text{ Kips} + 50.5 \text{ Kips} - 39.25 \text{ Kip} + P_{CD} = 0$

$P_{CD} = 11.25 \text{ Kip (T)}$

Note. We could have also used the FBD of the Right-Portion of Section 3-3



FBD- Right Portion of Section 3-3

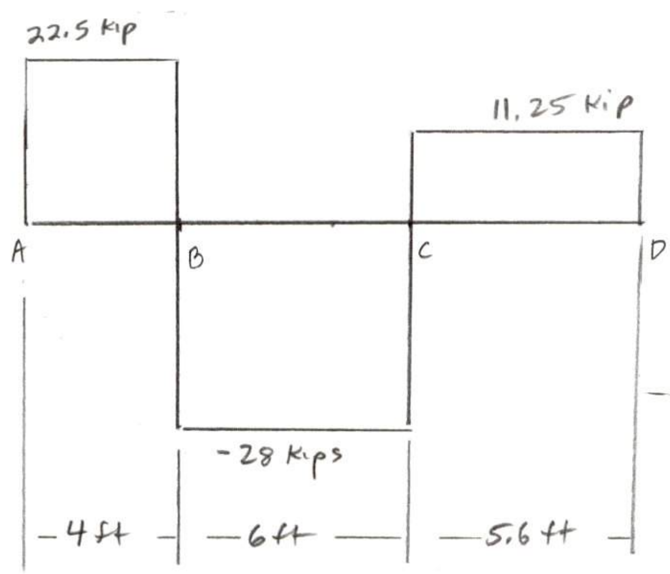
Equilibrium Equations

$(\Sigma F_x = 0)$

$$-P_{CD} + 11.25 \text{ kips} = 0$$

$$P_{CD} = 11.25 \text{ kips (T)}$$

Internal Axial Force Diagram



$E = 30{,}460$ ksi

$$A_{AB} = \frac{\pi (1.2 \text{ in})^2}{4} = 1.13 \text{ in}^2$$

$$A_{BC} = \frac{\pi (1.6 \text{ in})^2}{4} = 2.01 \text{ in}^2$$

$$A_{CD} = \frac{\pi (1.0 \text{ in})^2}{4} = 0.785 \text{ in}^2$$

$$\delta_{AB} = \frac{PL}{AE} = \frac{22.5 \text{ Kip } (4 \text{ ft})}{1.13 \text{ in}^2 \left(30{,}460 \frac{\text{Kip}}{\text{in}^2}\right)} = 0.002615 \text{ ft} \left(\frac{12 \text{ in}}{\text{ft}}\right) = 0.031377 \text{ in}$$

$$\delta_{BC} = \frac{-28 \text{ kip } (6 \text{ ft})}{2.01 \text{ in}^2 \left(30{,}460 \frac{\text{Kip}}{\text{in}^2}\right)} = -0.002744 \text{ ft} \left(\frac{12 \text{ in}}{\text{ft}}\right) = -0.03292 \text{ in}$$

$$\delta_{CD} = \frac{11.25 \text{ Kip } (5.6 \text{ ft})}{0.785 \text{ in}^2 \left(30{,}460 \frac{\text{Kip}}{\text{in}^2}\right)} = 0.002635 \text{ ft} \left(\frac{12 \text{ in}}{\text{ft}}\right) = 0.0316 \text{ in}$$

$$\delta_{AD} = 0.031386 \text{ in} - 0.03292 \text{ in} + 0.0316 \text{ in} = \underline{\underline{0.03 \text{ in}}}$$

10-14

A steel rod used in a control mechanism must transmit a tensile force of 2250 lb (10 kN) without exceeding an allowable stress of 21,756 psi (150 MPa) or stretching more than 0.04 in. (1 mm) per 3.3 ft (1 meter) of length. The modulus of elasticity is E= 30,460 ksi (210 GPa). Find the proper diameter of the bar.

Allowable Tensile Stress

$$\sigma_{allow} = 21{,}756 \text{ psi}$$

$$A = \frac{P}{\sigma_{allow}} = \frac{2250 \text{ lb}}{21{,}756 \frac{\text{lb}}{\text{in}^2}} = 0.1034 \text{ in}^2$$

Axial Deformation

$$\delta = \frac{PL}{AE} \Rightarrow A = \frac{PL}{\delta E}$$

$$= \frac{2250 \text{ lb}\left(\frac{\text{kip}}{1000 \text{ lb}}\right)(3.3 \text{ ft})\left(\frac{12 \text{ in}}{\text{ft}}\right)}{(0.04 \text{ in})\left(30{,}460 \frac{\text{kip}}{\text{in}^2}\right)}$$

$$= 0.073 \text{ in.}^2$$

The area for tensile stress is greater, stress controls

$$\frac{\pi d^2}{4} = 0.1034 \text{ in.}^2$$

$$d = \sqrt{\frac{4(0.1034 \text{ in.}^2)}{\pi}} = 0.3629 \text{ in.}$$

use, $\underline{\underline{d = 3/8 \text{ in.}}}$ (0.375 in)